Real Numbers and Operations 1.1.1

1 Astronomical Unit = 1.0 AU = 1.49×10^8 kilometers
1 Parsec = 3.26 Light years = 3×10^{18} centimeters = 206,265 AU
1 Watt = 10^7 ergs/sec
1 Star = 2×10^{33} grams

1 Yard = 36 inches	**1 meter = 39.37 inches**	**1 mile = 5,280 feet**
1 Liter = 1000 cm3	**1 inch = 2.54 centimeters**	**1 kilogram = 2.2 pounds**
1 Gallon = 3.78 Liters	**1 kilometer = 0.62 miles**	

For the unit conversion problems below, use a calculator and state your answers to two significant figures.

Problem 1 – Convert 11.3 square feet into square centimeters.

Problem 2 – Convert 250 cubic inches into cubic meters.

Problem 3 – Convert 1000 watts/meter^2 into watts/foot^2

Problem 4 – Convert 5 miles into kilometers.

Problem 5 – Convert 1 year into seconds.

Problem 6 – Convert 1 km/sec into parsecs per million years.

Problem 7 - A house is being fitted for solar panels. The roof measures 50 feet x 28 feet. The solar panels cost \$1.00/cm^2 and generate 0.03 watts/cm^2. A) What is the maximum electricity generation for the roof in kilowatts? B) How much would the solar panels cost to install? C) What would be the owners cost for the electricity in dollars per watt?

Problem 8 – A box of cereal measures 5 cm x 20 cm x 40 cm and contains 10,000 Froot Loops. What is the volume of a single Froot Loop in cubic millimeters?

Problem 9 – In city driving, a British 2002 Jaguar is advertised as having a gas mileage of 13.7 liters per 100 km, and a 2002 American Mustang has a mileage of 17 mpg. Which car gets the best gas mileage?

Problem 10 – The Space Shuttle used 800,000 gallons of rocket fuel to travel 400 km into space. If one gallon of rocket fuel has the same energy as 5 gallons of gasoline, what is the equivalent gas mileage of the Space Shuttle in gallons of gasoline per mile?

Answer Key

Problem 1 – 11.3 x (12 inches/foot)x(12 inches/foot) x (2.54 cm/1 inch)x(2.54 cm/1 inch) = 11,000 cm^2

Problem 2 – 250 $inch^3$ x $(2.54\ cm/inch)^3$ x $(1\ meter/100\ cm)^3$ = 0.0041 m^3

Problem 3 – 1000 $watts/meter^2$ x $(1\ meter/39.37\ inches)^2$ x $(12\ inches/foot)^2$ = 93 $watts/ft^2$

Problem 4 – 5 miles x (5280 feet/mile) x (12 inches/foot)x(2.54 cm/inch)x(1 meter/100 cm) x(1 km/1000 meters) = 8.1 km

Problem 5 – 1 year x (365 days/year)x(24 hours/day)x(60 minutes/hr)x(60 seconds/minute) = 32,000,000 seconds.

Problem 6 – 1.0 km/sec x (100000 cm/km)x(3.1 x 10^7 seconds/year) x (1.0 parsec/ 3.1 x 10^{18} cm) x (1,000,000 years/1 million years) = 1 parsec/million years

Problem 7 - A) Area = 50 feet x 28 feet = 1400 ft^2. Convert to cm^2: 1400 x $(12\ inch/foot)^2$ x $(2.54\ cm/1\ inch)^2$ = 1,300,642 cm^2. Maximum power = 1,300,642 cm^2 x 0.03 $watts/cm^2$ = 39.0 kilowatts. B) 1,300,642 cm^2 x \$1.00 $/cm^2$ = \$1.3 million C) \$1,300,000 / 39,000 watts = \$33 /watt.

Problem 8 – Volume of box = 5 x 20 x 40 = 4000 cm^3. This contains 10,000 Froot Loops, so each one has a volume of 4,000 cm^3/10,000 loops = 0.4 cm^3/Loop. Converting this into cubic millimeters: 0.4 cm^3 x $(10\ mm/1\ cm)^3$ = 400 mm^3/Loop.

Problem 9 – Convert both to kilometers per liter. Jaguar = 100 km/13.7 liters = 7.3 km/liter. Mustang = 17.0 x (1 km/0.62 miles) x(1 gallon/3.78 liters) = 7.3 km/liter. They both get similar gas mileage under city conditions.

Problem 10 – 400 km x (0.62 miles/km) = 248 miles. Equivalent gallons of gasoline = 800,000 gallons rocket fuel x (5 gallons gasoline/1 gallon rocket fuel) = 4,000,000 gallons gasoline, so the ‘mpg’ is 248 miles/4000000 = 0.000062 miles/gallon or 16,000 gallons/mile.
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Converting from one set of units (u-nuts, hence the squirrel photo to the left!) to another is something that scientists do every day. We have made this easier by adopting metric units wherever possible, and re-defining our standard units of measure so that they are compatible with the new metric units wherever possible.

In the western world, certain older units have been replaced by the modern ones, which are now adopted the world over. (see Wikipedia under 'English Units' for more examples).

Conversion Table:

4 Gallons = 1 Bucket	142.065 cubic cm = 1 Noggin
9 Gallons = 1 Firkin	1.296 grams = 1 Scruple
126 Gallons = 1 Butt	201.168 meters = 1 Furlong
34.07 Liters = 1 Firkin	14 days = 1 Fortnight
0.0685 Slugs = 1 Kilogram	

In the unit conversion problems below, use a calculator and give all answers to two significant figures.

Problem 1 - A typical aquarium holds 25 gallons of water. Convert this to A) Firkins; B) Liters, and C) Buckets.

Problem 2 - John weighs 7.2 Slugs, and Mary weighs 53 kilograms. Who weighs the most kilograms?

Problem 3 - The passenger volume of a car is about 5.4 cubic meters. How many Noggins can fit inside the car?

Problem 4 - Sven weighs 105 kilograms and finished a diet of pickled herring, losing 3.8 kilograms. A) How many Scruples did he lose? B) How many Scruples did he start out with?

Problem 5 - The density of water is 1.0 grams/cm^3. How many Scruples per Noggin is this?

Problem 6 - Evelyn finished the Diamond Man Marathon by walking 400 kilometers in 18 days. What was her average speed in Furlongs per Fortnight?

Answer Key:

Conversion Table:

4 Gallons = 1 Bucket	142.065 cubic centimeters = 1 Noggin
9 Gallons = 1 Firkin	1.296 grams = 1 Scruple
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34.07 Liters = 1 Firkin	14 days = 1 Fortnight
0.0685 Slugs = 1 Kilogram	

1) A typical aquarium holds 25 gallons of water. Convert this to

A) Firkins; 25 Gallons x (1 Firkin/9 Gallons) = **2.8 Firkins**

B) Liters, and 2.8 Firkins x (34.07 Liters/ 1 Firkin) = **95.0 Liters**

C) Buckets. 25 Gallons x (1 Bucket/4 gallons) = **6.2 Buckets.**

2) John weighs 7.2 Slugs, and Mary weighs 53 kilograms. Who weighs the most kilograms?

John = = 7.2 Slugs x (1 kg/0.0685 Slugs) = **110 kg so John weighs the most kgs.**

3) The passenger volume of a car is about 5.4 cubic meters. How many Noggins can fit inside the car?

5.4 cubic meters x (1,000,000 cubic cm/1 cubic meter) x(1 Noggin/142.065 cubic cm) = **38,000 Noggins!**

4) Sven weighs 105 kilograms and finished a diet of pickled herring, losing 3.8 kilograms.

A) How many Scruples did he lose? 3.8 kg x (1,000 gm/ 1kg) x (1 Scruple/1.296 grams) = **2,932 Scruples.**

B) How many Scruples did he start out with? 105 kg x(1,000 gm/1 kg) x 1 Scruple/1.296 grams) = **81,000 Scruples**

5) The density of water is 1.0 grams per cubic centimeter. How many Scruples per Noggin is this?

1 gram x (1 Scruple/1.296 grams) = 0.771 Scruples.

1 cubic centimeter x (1 Noggin/142.065 cubic cm) = 0.007 Noggins.

Dividing the two you get 0.771 Scruples/0.007 Noggins = **110 Scruples/Noggin.**

6) Evelyn finished the Diamond Man Marathon by walking 400 kilometers in 18 days. What was her average speed in Furlongs per Fortnight?

400 kilometers x (1,000 meters/ 1 km) x (1 Furlong/201 meters) = 1,990 Furlongs.

18 days x (1 Fortnight/14 days) = 1.28 Fortnights.

Dividing the two you get 1,990 Furlongs/1.28 Fortnights = **1,600 Furlongs/ fortnight.**

Stars are spread out through space at many different distances from our own Sun and from each other. In this problem, you will calculate the distances between some familiar stars using the 3-dimensional distance formula in Cartesian coordinates. Our own Sun is at the origin of this coordinate system, and all distances are given in light-years. The distance formula is given by the Pythagorean Theorem as:

$$d = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2}$$

Star	Distance from Sun	X	Y	Z	Distance from Polaris
Sun		0.0	0.0	0.0	
Sirius		-3.4	-3.1	7.3	
Alpha Centauri		-1.8	0.0	3.9	
Wolf 359		4.0	4.3	5.1	
Procyon		-0.9	5.6	-9.9	
Polaris		99.6	28.2	376.0	0.0
Arcturus		32.8	9.1	11.8	
Tau Ceti		-6.9	-8.6	2.5	
HD 209458		-94.1	-120.5	5.2	
Zubenelgenubi		64.6	-22.0	23.0	

Problem 1 - What are the distances of these stars from the Sun in light-years to two significant figures?

Problem 2 - If you moved to the North Star, Polaris, how far would the Sun and other stars be from you to two significant figures? Enter the answer in the table above.

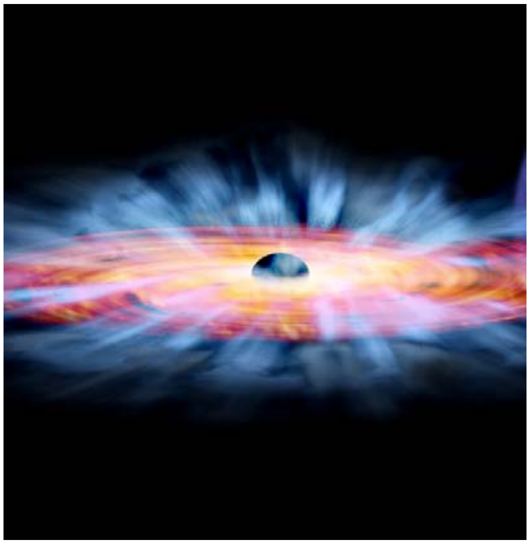
Problem 3 - Which of these stars is the closest to Polaris?

Star	Distance from Sun	X	Y	Z	Distance from Polaris
Sun	**0.0**	0.0	0.0	0.0	**390**
Sirius	**8.7**	-3.4	-3.1	7.3	**380**
Alpha Centauri	**4.3**	-1.8	0.0	3.9	**390**
Wolf 359	**7.8**	4.0	4.3	5.1	**380**
Procyon	**11.0**	-0.9	5.6	-9.9	**400**
Polaris	**390**	99.6	28.2	376.0	**0**
Arcturus	**36**	32.8	9.1	11.8	**370**
Tau Ceti	**11.0**	-6.9	-8.6	2.5	**390**
HD 209458	**150**	-94.1	-120.5	5.2	**400**
Zubenelgenubi	**72**	64.6	-22.0	23.0	**360**

Problem 1 - : What are the distances of these stars from the Sun in light-years to two significant figures? **Answer:** Use the formula provided with $X_1=0$, $y_1=0$ and $z_1 = 0$. Example for Sirius
where $x_2 = -3.4$, $y_2 = -3.1$ and $z_2=7.3$ yields,
$D = ((-3.4)^2 + (-3.1)^2 + (7.3)^2)^{1/2}$
= **8.7 light-years.**

Problem 2 - If you moved to the North Star, Polaris, how far would the Sun and other stars be from you? Enter the answer in the table. **Answer:** To do this, students select the new origin at Polaris and fix x_1 = **99.6**, y_1=**28.2** and z_1 = **376.0** in the distance formula. They then insert the X, Y and Z coordinates for the other stars and compute the distance. Example, for the Sun, the distance will be 390 light years, because that is how far Polaris is from the Sun. For HD 209458, the distance formula gives
$D = ((-94.1 - \mathbf{99.6})^2 + (-120.5 - \mathbf{28.2})^2 + (5.2 - 376)^2)^{1/2}$
$= (37519 + 22111 + 137492)^{1/2}$
= **440 light years.**

Problem 3 - Which of these stars is the closest to Polaris? **Answer:** Zubenelgenubi!



A tidal force is a difference in the strength of gravity between two points. The gravitational field of the moon produces a tidal force across the diameter of Earth, which causes the Earth to deform. It also raises tides of several meters in the solid Earth, and larger tides in the liquid oceans. If the satellite gets too close it can be tidally disrupted. The artistic image to the left shows what tidal disruption could be like for an unlucky moon.

A human falling into a black hole will also experience tidal forces. In most cases these will be lethal! The difference in gravitational force between the head and feet could be so intense that a person would literally be pulled apart! Some physicists have termed this process spaghettification!

$$a = \frac{2GMd}{R^3}$$

Problem 1 - The equation lets us calculate the tidal acceleration, **a**, across a body with a length of **d**. The tidal acceleration between your head and feet is given by the above formula. For M = the mass of Earth (5.9×10^{27} grams), R = the radius of Earth (6.4×10^{8} cm) and the constant of gravity whose value is G = 6.67×10^{-8} dynes cm^2/gm^2 calculate the tidal acceleration, a, if a typical human height is d = 200 centimeters.

Problem 2 - What is the tidal acceleration across the full diameter of Earth?

Problem 3 - A stellar-mass black hole has the mass of the sun (1.9×10^{33} grams), and a radius of 2.9 kilometers. A) At a distance of 100 kilometers, what would be the tidal acceleration across a human for d=200 cm? B) If the acceleration of gravity at Earth's surface is 980 cm/sec^2, would the unlucky human traveler be spaghettified near a stellar-mass black hole?

Problem 4 - A supermassive black hole has 100 million times the mass of the sun (1.9×10^{33} grams), and a radius of 295 million kilometers. What would be the tidal acceleration across a human with d = 2 meters, at a distance of 100 kilometers from the event horizon of the supermassive black hole?

Problem 5 - Which black hole could a human enter without being spagettified?

Answer Key:

Problem 1 - The equation lets us calculate the tidal acceleration, **a**, across a body with a length of **d**. The tidal acceleration between your head and feet is given by the above formula. For M = the mass of Earth (5.9×10^{27} grams), R = the radius of Earth (6.4×10^{8} cm) and the constant of gravity whose value is G = 6.67×10^{-8} dynes cm^2/gm^2 calculate the tidal acceleration, a, if d = 2 meters.

Answer: $a = [2 \times (6.67 \times 10^{-8}) \times (5.9 \times 10^{27}) \times 200] / (6.4 \times 10^{8})^3$

$= 0.000003 \times (200)$

= 0.0006 cm/sec^2

Problem 2 - What is the tidal acceleration across the full diameter of Earth?
Answer: $d = 1.28 \times 10^{9}$ cm, so $a = 0.000003 \times 1.28 \times 10^{9}$ = **3,800 cm/sec^2**

Problem 3 - A stellar-mass black hole has the mass of the sun (1.9×10^{33} grams), and a radius of 2.9 kilometers. A) What would be the tidal acceleration across a human at a distance of 100 kilometers? B) Would a human be spaghettified?

Answer: A) $a = 2 \times (6.67 \times 10^{-8}) \times (1.9 \times 10^{33}) \times 200 / (1.0 \times 10^{7})^3$

= **51,000,000 cm/sec^2**

B) Yes, this is equal to 51,000,000/979 = 52,000 times the acceleration of gravity, and a human would be pulled apart and 'spaghettified'

Problem 4 - A supermassive black hole has 100 million times the mass of the sun (1.9×10^{33} grams), and an event horizon radius of 295 million kilometers. What would be the tidal acceleration across a d=2 meter human at a distance of 100 kilometers from the event horizon of the supermassive black hole?

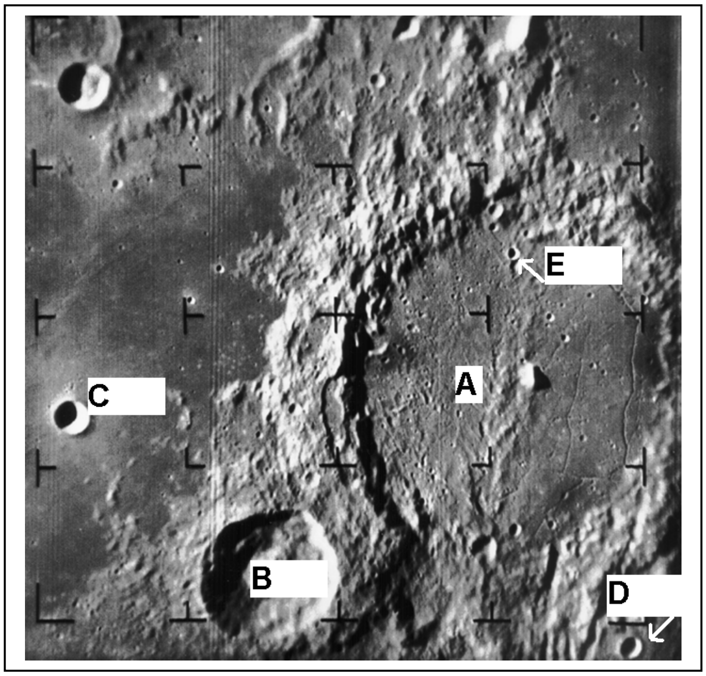
Answer: $a = 2 \times (6.67 \times 10^{-8}) \times (1.9 \times 10^{41}) \times 200 / (2.95 \times 10^{13})^3$

= 0.00020 cm/sec^2

Note that R + 2 meters is essentially R if R = 295 million kilometers.

Problem 5 - Which black hole could a human enter without being spaghettified?
Answer: The supermassive black hole, because the tidal force is far less than what a human normally experiences on the surface of Earth. That raises the question whether as a space traveler, you could find yourself trapped by a supermassive black hole unless you knew exactly what its size was before hand. You would have no physical sensation of having crossed over the black hole's Event Horizon before it was too late.

Algebraic Expressions and Models



Have you ever wondered how much energy it takes to create a crater on the Moon. Physicists have worked on this problem for many years using simulations, and even measuring craters created during early hydrogen bomb tests in the 1950's and 1960's. One approximate result is a formula that looks like this:

$$\mathbf{E = 4.0 \times 10^{15} D^3 \text{ Joules.}}$$

where D is the crater diameter in multiples of 1 kilometer.

As a reference point, a nuclear bomb with a yield of one-megaton of TNT produces 4.0×10^{15} Joules of energy!

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of the indicated craters in the picture.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified.

Note: To get a better sense of scale, the table below gives some equivalent energies for famous historical events:

Table of impact energies

Event	Equivalent Energy (TNT)
Cretaceous Impactor	100,000,000,000 megatons
Valdiva Volcano, Chile 1960	178,000 megatons
San Francisco Earthquake 1909	600 megatons
Hurricane Katrina 2005	300 megatons
Krakatoa Volcano 1883	200 megatons
Tsunami 2004	100 megatons
Mount St. Helens Volcano 1980	25 megatons

Answer Key

Problem 1 - To make the formula more 'real', convert the units of Joules into an equivalent number of one-megaton nuclear bombs.

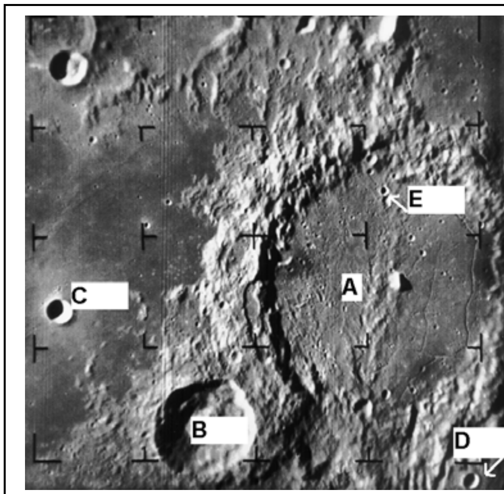
Answer: $E = 4.0 \times 10^{15} D^3$ Joules x (1 megaton TNT/4.0×10^{15} Joules)

$E = 1.0\ D^3$ megatons of TNT

Problem 2 - The photograph above was taken in 1965 by NASA's Ranger 9 spacecraft of the large crater Alphonsis. The width of the image above is 183 kilometers. With a millimeter ruler, determine the diameters, in kilometers, of a range of craters in the picture. Answer: The width of the image is 92 mm, so the scale is 183/92 = 2.0 km/mm. See figure below for some typical examples: See column 3 in the table below for actual crater diameters.

Problem 3 - Use the formula from Problem 1 to determine the energy needed to create the craters you identified. Answer: See the table below, column 4. Crater A is called Alphonsis. Note: No single formula works for all possible scales and conditions. The impact energy formula only provides an estimate for lunar impact energy because it was originally designed to work for terrestrial impact craters created under Earth's gravity and bedrock conditions. Lunar gravity and bedrock conditions are somewhat different and lead to different energy estimates. The formula will not work for laboratory experiments such as dropping pebbles onto sand or flour. The formula is also likely to be inaccurate for very small craters less than 10 meters, or very large craters greatly exceeding the sizes created by nuclear weapons. (e.g. 1 kilometer).

Crater	Size (mm)	Diameter (km)	Energy (Megatons)
A	50	**100**	**1,000,000**
B	20	**40**	**64,000**
C	5	**10**	**1,000**
D	3	**6**	**216**
E	1	**2**	**8**



Algebraic Expressions and Models

Symbol	Name	Value
c	Speed of light	2.9979×10^{10} cm/sec
h	Planck's constant	6.6262×10^{-27} erg sec
m	Electron mass	9.1095×10^{-28} gms
e	Electron charge	4.80325×10^{-10} esu
G	Gravitation constant	6.6732×10^{-8} dyn cm^{2} gm^{-2}
M	Proton mass	1.6726×10^{-24} gms

Also use $\pi = 3.1415926$

Although there are only a dozen fundamental physical constants of Nature, they can be combined to define many additional basic constants in physics, chemistry and astronomy.

In this exercise, you will evaluate a few of these 'secondary' constants to three significant figure accuracy using a calculator and the defined values in the table.

Problem 1 - Black Hole Entropy Constant: $\frac{c^3}{2hG}$

Problem 2 - Gravitational Radiation Constant: $\frac{32}{5}\frac{G^5}{c^{10}}$

Problem 3 - Thomas-Fermi Constant: $\frac{324}{175}\left(\frac{4}{9\pi}\right)^{2/3}$

Problem 4 - Thompson Scattering Cross-section: $\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$

Problem 5 - Stark Line Limit: $\frac{16\pi^4 m^2 e^4}{h^4 M^5}$

Problem 6 - Bremstrahlung Radiation Constant: $\frac{32\pi^2 e^6}{3(2\pi)^{1/2} m^3 c}$

Problem 7 - Photoionization Constant: $\frac{32\pi^2 e^6 (2\pi^2 e^4 m)}{3^{3/2} h^3}$

Answer Key

Method 1: Key-in to a calculator all the constants with their values as given to all indicated significant figures, write down final calculator answer, and round to three significant figures.

Method 2: Round all physical constants to 4 significant figures, key-in these values on the calculator, then round final calculator answer to 3 significant figures.

Note: When you work with numbers in scientific notation, example 1.23×10^5, the leading number '1.23' has 3 significant figures, but 1.23000 has 6 significant figures if the '000' are actually measured to be '000', otherwise they are just non-significant placeholders.

Also, you cannot have a final answer in a calculation that has more significant figures than the smallest significant figure number in the set. For example, 6.25*5.1 which a calculator would render as 31.875 is 'only good' to 2 significant figures (determined from the number 5.1) so the correct, rounded, answer is 32.

Problem	Method 1	Method 2
1	3.05×10^{64}	3.05×10^{64}
2	1.44×10^{-140}	1.44×10^{-140}
3	5.03×10^{-1}	5.03×10^{-1}
4	6.65×10^{-25}	6.64×10^{-25}
5	2.73×10^{135}	2.73×10^{135}
6	2.28×10^{16}	2.27×10^{16}
7	2.46×10^{-39}	2.46×10^{-39}

Note Problem 4 and 6 give slightly different results.

Problem 1: Method 1 answer 3.8784/1.7042 = 2.27578 or 2.28
Method 2 answer 3.878/1.705 = 2.274 = 2.27

Problem 4: Method 1 answer 1.3378/2.0108 = 0.6653 = 0.665
Method 2 answer 1.338/2.014 = 0.6642 = 0.664

Algebraic Expressions and Models

A magnetic field is more complicated in shape than a gravitational field because magnetic fields have a property called 'polarity'. All magnets have a North and South magnetic pole, and depending on where you are in the space near a magnet, the force you feel will be different than for gravity. The strength of the magnetic field along each of the three directions in space (X, Y and Z) is given by the formulas:

$$B_x = \frac{3 x z M}{r^5}$$

$$B_y = \frac{3 y z M}{r^5}$$

$$B_z = \frac{(3 z^2 - r^2) M}{r^5}$$

The variables X, Y and Z represent the distance to a point in space in terms of the radius of Earth. For example, 'X = 2.4' means a physical distance of 2.4 times the radius of Earth or (2.4 x 6378 km) = 15,000 kilometers. Any point in space near Earth can be described by its address (X, Y, Z). The variable r is the distance from the point at (X, Y, Z) to the center of Earth in units of the radius of Earth. **M** is a constant equal to 31,000 nanoTeslas.

The formula for the three quantities Bx, By and Bz gives their strengths along each of the three directions in space, in units of nanoTeslas (nT) – a measure of magnetic strength.

Problem 1 - Evaluate these three equations to two significant figures at the orbit of communications satellites for the case where x = 7.0, y = 0.0, z = 0.0 and r = 7.0

Problem 2 - Evaluate these three equations to two significant figures in the Van Allen Belts for the case where x = 0.38, y = 0.19, z = 1.73 and r = 3.0

Problem 3 - Evaluate these three equations at the distance of the Moon to two significant figures for the case where x = 0.0, y = 48.0, z = 36 and r = 60.0

Problem 4 - Use the Pythagorean Theorem in 3-dimensions to determine, to two significant figures, the absolute magnitude of Earth's magnetic field for each of the problems 1, 2 and 3.

Answer Key

Problem 1 - For x = 7.0, y = 0.0, z = 0.0 and r = 7.0

$Bx = 3 (7.0) (0.0) (31,000)/(7.0)^5 =$ **0.0 nT**

$By = 3 (0.0) (0.0) (31,000) / (7.0)^5 =$ **0.0 nT**

$Bz = [3(0.0)^2 – (7.0)^2](31,000) / (7.0)^5$
$= - (31,000)(7.0)^2/(7.0)^5$
$= - 1,519,000 / 16807$
$=$ **- 90 nT**

Problem 2 - For x = 0.38, y = 0.19, z = 1.73 and r = 3.0
$Bx = 3 (0.38) (1.73) (31,000)/(3.0)^5 =$ **+250 nT**

$By = 3 (0.19) (1.73) (31,000) / (3.0)^5 =$ **+130 nT**

$Bz = [3(1.73)^2 – (3.0)^2) (31,000)/ (3.0)^5$
$= (-0.021)(31000)/243$
$=$ **- 2.7 nT**

Problem 3 - For x = 0.0, y = 48.0, z = 36 and r = 60.0
$Bx = 3 (0.0) (36) (31,000)/(60)^5 =$ **0.0 nT**

$By = 3 (48.0) (36) (31,000) / (60)^5 =$ **0.21 nT**

$Bz = [3(36)^2 – (60)^2] (31,000) / (60)^5$
$= (288)(31,000)/(7,776,000,000)$
$=$ **0.0011 nT**

Problem 4 - Use the Pythagorean Theorem in 3-dimensions to determine the total strength of Earth's magnetic field for problems 1, 2 and 3.

1) $B = (Bx^2 + By^2 + Bz^2)^{1/2} = ((-90)^2)^{1/2} =$ **90 nT** at communications satellite orbit.

2) $B = ((251)^2 +(126)^2 +(-2.7)^2)^{1/2} =$ **280 nT** at Van Allen belts

3) $B = ((0.0)^2 +(0.21)^2 +(0.0011)^2)^{1/2} =$ **0.21 nT** at the Moon

Potential energy is the energy that a body possesses due to its **location** in space, while kinetic energy is the energy that it has depending on its **speed** through space. For locations within a few hundred kilometers of Earth's surface, neglecting air resistance, and for speeds that are small compared to that of light, we have the two energy formulae:

$$P.E = mgh \qquad K.E = \frac{1}{2}mV^2$$

where g is the acceleration of gravity near Earth's surface and has a value of 9.8 meters/sec^2. If we use units of mass, **m**, in kilograms, height above the ground, **h**, in meters, and the body's speed, **V**, in meters/sec, the units of energy (P.E and K.E.) are Joules.

As a baseball, a coasting rocket, or a stone dropped from a bridge moves along its trajectory back to the ground, it is constantly exchanging, joule by joule, potential energy for kinetic energy. Before it falls, its energy is 100% P.E, while in the instant just before it lands, its energy is 100% K.E.

Problem 1 - A baseball with m = 0.145 kilograms falls from the top of its arc to the ground; a distance of 100 meters. A) What was its K.E., in Joules, at the top of its arc? B) To two significant figures, what was the baseball's P.E. in Joules at the top of the arc?

Problem 2 - The Ares 1-X capsule had a mass of 5,000 kilograms. If the capsule fell 45 kilometers from the top of its trajectory 'arc', how much kinetic energy did it have at the moment of impact with the ground?

Problem 3 - Suppose that the baseball in Problem 1 was dropped from the same height as the Ares 1-X capsule. What would its K.E. be at the moment of impact?

Problem 4 - From the formula for K.E. and your answers to Problems 2 and 3, in meters/sec to two significant figures; A) What was the speed of the baseball when it hit the ground? B) What was the speed of the Ares 1-X capsule when it landed? C) Discuss how your answers do not seem to make 'common sense'.

Problem 1 - A baseball with m = 0.145 kilograms falls from the top of its arc to the ground; a distance of 100 meters. A) What was its K.E., in joules, at the top of its arc? B) To two significant figures, what was the baseball's P.E. in joules at the top of the arc?
Answer: A) **K.E = 0** B) P.E. = mgh = (0.145) x (9.8) x (100) = **140 joules.**

Problem 2 - The Ares 1-X capsule had a mass of 5,000 kilograms. If the capsule fell 45 kilometers from the top of its trajectory 'arc', how much kinetic energy did it have at the moment of impact with the ground?
Answer: At the ground, the capsule has exchanged all of its potential energy for 100% kinetic energy so K.E. = P.E. = mgh. Then K.E. = (5,000 kg) x (9.8) x (45,000 meters) = **2.2 billion Joules for the Ares 1-X capsule.**

Problem 3 - Suppose that the baseball in Problem 1 was dropped from the same height as the Ares 1-X capsule. What would its K.E. be at the moment of impact?
Answer: its K.E. would equal 100% of its original P.E. so K.E = mgh = (0.145 kg) x (9.8) x (45,000 meters) = **64,000 Joules for the baseball.**

Problem 4 - From the formula for K.E. and your answers to Problems 2 and 3, in meters/sec to two significant figures; A) What was the speed of the baseball when it hit the ground? B) What was the speed of the Ares 1-X capsule when it landed? C) Discuss how your answers do not seem to make 'common sense'.

Answer; A) Baseball: K.E. = $1/2\ m\ V^2$

$$V = (2E/m)^{1/2}$$
$$= (2(64000)/0.145))^{1/2}$$
$$\mathbf{= 940\ meters/sec.}$$

B) Capsule: $V = (2\ (2{,}200{,}000{,}000)/5{,}000)^{1/2}$

$$\mathbf{= 940\ meters/sec}$$

C) The misconception is that our intuition suggests that the much heavier Ares 1-X capsule should have struck the ground at a far-faster speed!

Planets have been spotted orbiting hundreds of nearby stars. The temperature of the surface of the planet depends on how far the planet is located from its star, and on the star's luminosity. The temperature of the planet, neglecting its atmosphere, will be about

$$T=273\left(\frac{(1-A)L}{D^2}\right)^{1/4}$$

where A is the reflectivity (albedo) of the planet, L is the luminosity of its star in multiples of the sun's power, and D is the distance between the planet and the star in Astronomical Units (AU). The resulting temperature will be in units of Kelvin. (i.e. 0^{o} Celsius = +273 K, and Absolute Zero is defined as 0 K)

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4?

Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of the star were increased 1000 times and all other quantities remained the same?

Problem 3 - The recently discovered planet CoRoT-7b (see artist's impression above, from ESA press release), orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures, to two significant figures, of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	
Iron Oxide	Mars	0.16	
Water+Land	Earth	0.40	
Gas	Jupiter	0.70	

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4? **Answer: $T = 273\ (0.6)^{1/4} = 240$ K**

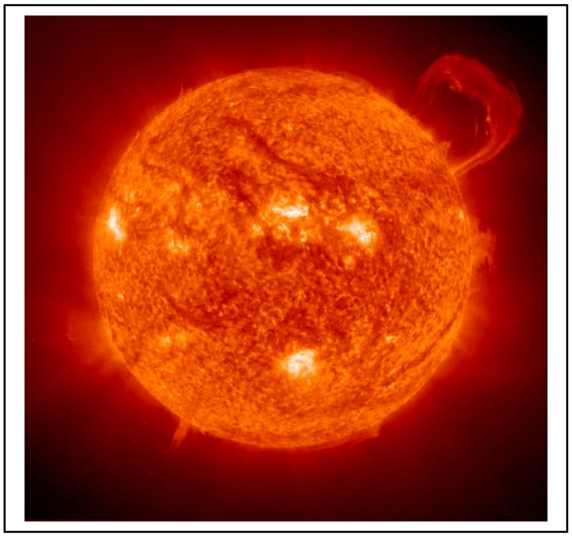
Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of the star were increased 1000 times and all other quantities remained the same? Answer: From the formula, T = 240 and L = 1000 so
$240 = 273(0.6 \times 1000/D^2)^{1/4}$ and so **D = 32 AU**. This is about near the orbit of Neptune!

Problem 3 - The recently discovered planet CoRoT-7b orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	**1900**
Iron Oxide	Mars	0.16	**1800**
Water+Land	Earth	0.40	**1700**
Gas	Jupiter	0.70	**1400**

Example: For an albedo similar to that of our Moon:
$T = 273 * ((1-0.06)*0.71/(0.017)^2)^{.25}$
= **1,900 Kelvin**

Note: To demonstrate the concept of Significant Figures, the values for L, D and A are given to 2 significant figures, so the answers should be rounded to 1900, 1800, 1700 and 1400 respectively



Detailed mathematical models of the interior of the sun are based on astronomical observations and our knowledge of the physics of stars. These models allow us to explore many aspects of how the sun 'works' that are permanently hidden from view.

The Standard Model of the sun, created by astrophysicists during the last 50 years, allows us to investigate many separate properties. One of these is the density of the heated gas throughout the interior. The function below gives a best-fit formula, D(x) for the density (in grams/cm^3) from the core (x=0) to the surface (x=1) and points in-between.

$$D(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$$

For example, at a radius 30% of the way to the surface, x = 0.3 and so D(x=0.3) = 14.5 grams/cm^3.

Problem 1 - What is the estimated core density of the sun?

Problem 2 - To the nearest 1% of the radius of the sun, at what radius does the density of the sun fall to 50% of its core density at x=0? (Hint: Use a graphing calculator and estimate x to 0.01)

Problem 3 - To three significant figures, what is the estimated density of the sun near its surface at x=0.9 using this polynomial approximation?

Answer Key

Problem 1 - Answer; At the core, x=0, do D(0) = 155 grams/cm^3.

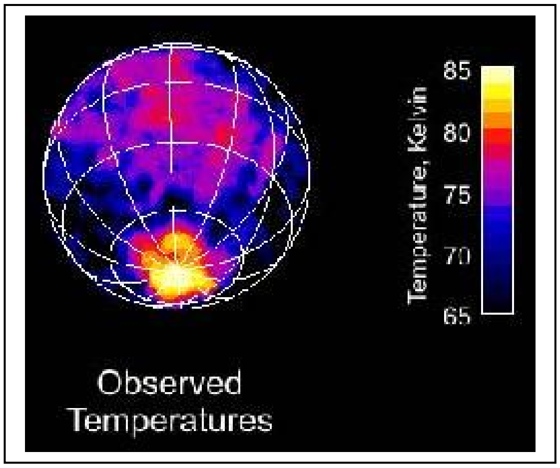
Problem 2 - Answer: We want D(x) = 155/2 = 77.5 gm/cm^3. Use a graphing calculator, or an Excell spreadsheet, to plot D(x) and slide the cursor along the curve until D(x) = 77.5, then read out the value of x. The relevant portion of D(x) is shown in the table below:

X	D(x)
0.08	94.87
0.09	88.77
0.1	82.96
0.11	**77.43**
0.12	72.16
0.13	67.16
0.14	62.41

Problem 3 - Answer: At x=0.9 (i.e., a distance of 90% of the radius of the sun from the center).
$D(0.9) = 519(0.9)^4 - 1630(0.9)^3 + 1844(0.9)^2 - 889(0.9) + 155$
$D(0.9) = 340.516 - 1188.27 + 1493.64 - 800.10 + 155.00$
D(0.9) = 0.786 gm/cm^3.

Note: The density of water id 1.0 gm/cm^3 so this solar material would 'float' on water!

Equilibrium Temperature



This temperature map of the satellite Enceladus was created from the infrared data of NASA's Cassini spacecraft.

As a body absorbs energy falling on its surface, it also emits energy back into space. When the 'energy in' matches the 'energy out' the body maintains a constant 'equilibrium' temperature.

If the body absorbs 100% of the energy falling on it, the relationship between the absorbed energy in watts/meter^2, F, and the equilibrium temperature measured in degrees Kelvin, T, is given by

$$F = 5.7 \times 10^{-8}\ T^4$$

Problem 1 - A human body has a surface area of 2 meters^2, and is at a temperature of 98.6^o F (310 Kelvin). What is the total emitted power of a human in watts?

Problem 2 - Sunlight falling on a body at Earth delivers 1,357 watts/meter^2. What would be the temperature, in Kelvin and Celsius, of the body if all of this solar energy flux were completely absorbed by the body?

Problem 3 - A 2000-Kelvin lava flow is 10 meters wide and 100 meters long. What is the total thermal power output of this heated rock in megawatts?

Problem 4 - A 2 square-meter piece of aluminum is painted so that it absorbs only 10% of the solar energy falling on it (Albedo = 0.9). If the aluminum panel is on the outside of the International Space Station, and the solar flux in space is 1,357 watts/meter^2, what will be the equilibrium temperature, in Kelvins, Celsius and Fahrenheit, of the panel in full sunlight where the conversion formulae are: C = K - 273 and F = 9/5C + 32 ?

Answer Key

Problem 1 - A human body has a surface area of 2 meters^2, and is at a temperature of 98.6 F (310 Kelvin). What is the total emitted power of a human in watts?

Answer: $F = 5.7 \times 10^{-8} (310)^4 = 526$ watts/meter^2
Then P = F x area
= 526 x 2
= 1,100 watts.

Problem 2 - Sunlight falling on a body at Earth delivers 1,357 watts/meter^2. What would be the temperature, in Kelvin and Celsius, of the body if all of this solar energy flux were completely absorbed by the body?

Answer: $1,357 = 5.7 \times 10^{-8} T^4$ so
T = 393 Kelvin
T = 393 - 273
= 120 Celsius.

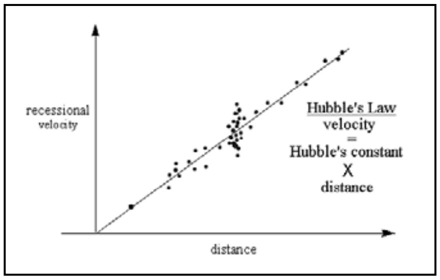
Problem 3 - A 2000-Kelvin lava flow is 10 meters wide and 100 meters long. What is the total thermal power output of this heated rock in megawatts?

Answer: $F = 5.7 \times 10^{-8} (2000)^4 = 912,000$ Watts/meter^2. The area is 1000 meters^2, so the power is 912,000 x 1000 **= 910 megawatts!**

Problem 4 - A 2 square-meter piece of aluminum is painted so that it absorbs only 10% of the solar energy falling on it (Albedo = 0.9). If the aluminum panel is on the outside of the International Space Station, and the solar flux in space is 1,357 watts/meter^2, what will be the equilibrium temperature, in Kelvins, Celsius and Fahrenheit, of the panel in full sunlight where the conversion formulae are: C = K - 273 and F = 9/5C + 32 ?

Answer: $F = (0.2 \times 1,357) = 5.7 \times 10^{-8} T^4$
so **T(K) = 262 K**
T(C) = 262 - 273 = **-11^{o}C**
T(F) = 9/5(-11) + 32 = **+12^o F**.

Solving Linear Equations



Calculations involving a single variable come up in many different ways in astronomy. One way is through the relationship between a galaxy's speed and its distance, which is known as Hubble's Law. Here are some more applications for you to solve!

1 – The blast wave from a solar storm traveled 150 million kilometers to Earth in 48 hours. Solve the equation 150,000,000 = 48 V to find the speed of the storm, V, in kilometers per hour.

2– A parsec equals 3.26 light years. If its distance is 4.3 light years, solve the equation 4.3 = 3.26D to find the distance to the star Alpha Centauri in parsecs.

3 – Hubble's Law states that distant galaxies move away from the Milky Way, 75 kilometers/sec faster for every 1 million parsecs of distance. Solve the equation, V = 75 D to find the speed of the galaxy NGC 4261 located 41 million parsecs away

4 – Convert the temperature at the surface of the sun, 9,900 degrees Fahrenheit to an equivalent temperature in Kelvin units, T, by using T = (F + 459) x 5/9

Answer Key

1 – The blast wave from a solar storm traveled 150 million kilometers in 48 hours. Solve the equation 150,000,000 = 48 V to find the speed of the storm, V, in kilometers per hour.
Answer: 150,000,000/48 = V so V = 3,125,000 kilometers/hour.

2 – A parsec equals 3.26 light years. Solve the equation 4.3 = 3.26D to find the distance to the star Alpha Centauri in parsecs, D, if its distance is 4.3 light years.
Answer: D = 4.3/3.26 = 1.3 parsecs.

3 – Hubble's Law states that distant galaxies move away from the Milky Way, 75 kilometers/sec faster for every 1 million parsecs of distance. V = 75 x D. Solve the equation to find the speed of the galaxy NGC 4261 located D = 41 million parsecs away
Answer: V = 75 x 41 so V = 3,075 kilometers/sec.

4 – Convert the temperature at the surface of the sun, 9,900 degrees Fahrenheit (F) to an equivalent temperature in Kelvin units, T, by using T = (F + 459) x 5/9
Answer: T = (F + 459) x 5/9 so T = (9,900 + 459) x 5/9 = 5,755 Kelvins

Rewriting Equations and Formulas

On July 19, 1969 the Apollo-11 Command Service Module and LEM entered lunar orbit.

The time required to travel once around in the orbit is called the orbit period, which was 2.0 hours, at a distance of 1,737 kilometers from the lunar center.

Believe it or not, you can use these two pieces of information to determine the mass of the moon. Here's how it's done!

Problem 1 - Assume that Apollo-11 went into a circular orbit, and that the inward gravitational acceleration by the moon on the capsule, Fg, exactly balances the outward centrifugal acceleration, Fc. Solve Fc = Fg for the mass of the moon, M, in terms of V, R and the constant of gravity, G, given that:

$$F_g = \frac{G\,M\,m}{R^2} \qquad F_c = \frac{m\,V^2}{R}$$

Problem 2 - By using the fact that for circular motion, $V = 2\pi R / T$, re-express your answer to Problem 1 in terms of R, T and M.

Problem 3 - Given that $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-2}$, R = 1,737 kilometers and T = 2 hours, calculate the mass of the moon, M, in kilograms!

Problem 4 - The mass of Earth is $M = 5.97 \times 10^{24}$ kilograms. What is the ratio of the moon's mass, derived in Problem 3, to Earth's mass?

Answer Key

Problem 1 - From Fg = Fc, and a little algebra to simplify and cancel terms, you get:

$$M = \frac{RV^2}{G}$$

Problem 2 - Substitute 2 π R / T for V and with little algebra to simplify and cancel terms, you get :

$$\boxed{\mathbf{M = \frac{4\pi^2 R^3}{G\ T^2}}}$$

Problem 3 - First convert all units to meters and seconds: R = 1.737×10^6 meters and T = 7,200 seconds. Then substitute values into the above equation:

$M = 4 \times (3.14)^2 \times (1.737 \times 10^6)^3 / (6.67 \times 10^{-11} \times (7200)^2))$

$M = (39.44 \times 5.24 \times 10^{18}) / (3.46 \times 10^{-3})$

M = $\mathbf{6.00 \times 10^{22}}$ **kilograms**

More accurate measurements, allowing for the influence of Earth's gravity and careful timing of orbital periods, actually yield 7.4×10^{22} kilograms.

Problem 4 - The ratio of the masses is 5.97×10^{22} kilograms / 5.97×10^{24} kilograms which equals **1/100**. The actual mass ratio is 1 / 80.

Rewriting Equations and Formulas

$$Fg = \frac{G M m}{R^2}$$

$$Fc = \frac{m V^2}{R}$$

$$V = \frac{2 \pi R}{T}$$

One of the neatest things in astronomy is being able to figure out the mass of a distant object, without having to 'go there'. Astronomers do this by employing a very simple technique. It depends only on measuring the separation and period of a pair of bodies orbiting each other. In fact, Sir Issac Newton showed us how to do this over 300 years ago!

Imagine a massive body such as a star, and around it there is a small planet in orbit. We know that the force of gravity, **Fg**, of the star will be pulling the planet inwards, but there will also be a centrifugal force, **Fc**, pushing the planet outwards.

This is because the planet is traveling at a particular speed, **V**, in its orbit. When the force of gravity and the centrifugal force on the planet are exactly equal so that **Fg = Fc**, the planet will travel in a circular path around the star with the star exactly at the center of the orbit.

Problem 1) Use the three equations above to derive the mass of the primary body, M, given the period, T, and radius, R, of the companion's circular orbit.

Problem 2) Use the formula $\mathbf{M = 4 \pi^2 R^3 / (G\ T^2)}$ where G = $\mathbf{6.6726 \times 10^{-11}\ m^3\ kg^{-1}\ sec^{-2}}$ and M is the mass of the primary in kilograms, R is the orbit radius in meters and T is the orbit period in seconds, to find the masses of the primary bodies in the table below to two significant figures. (Note: Make sure all units are in meters and seconds first! 1 light year = 9.5 trillion kilometers)

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	
Earth	Moon	27.3 days	385,000 km	
Jupiter	Callisto	16.7 days	1.9 million km	
Pluto	Charon	6.38 days	17,530 km	
Mars	Phobos	7.6 hrs	9,400 km	
Sun	Earth	365 days	149 million km	
Sun	Neptune	163.7 yrs	4.5 million km	
Sirius A	Sirius B	50.1 yrs	20 AU	
Polaris A	Polaris B	30.5 yrs	290 million miles	
Milky Way	Sun	225 million yrs	26,000 light years	

Answer Key

Problem 1: Answer

$$\frac{GMm}{R^2} = \frac{mV^2}{R}$$ cancel 'm' on both sides and re-arrange to solve for M

$$M = \frac{RV^2}{G}$$ now use the definition for V to eliminate V from the equation

$$M = \frac{R}{G}\left(\frac{2\pi R}{T}\right)^2$$ and simplify to get the required equation:

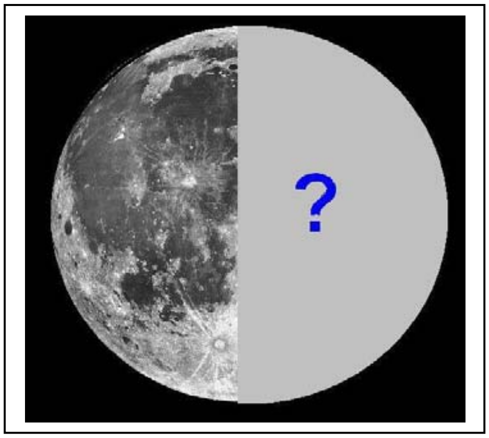
$$M = \frac{4\pi^2 R^3}{GT^2}$$

Problem 2:

Primary	Companion	Period	Orbit Radius	Mass of Primary
Earth	Communications satellite	24 hrs	42,300 km	**6.1 x 10^{24} kg**
Earth	Moon	27.3 days	385,000 km	**6.1 x 10^{24} kg**
Jupiter	Callisto	16.7 days	1.9 million km	**1.9 x 10^{27} kg**
Pluto	Charon	6.38 days	17,530 km	**1.3 x 10^{22} kg**
Mars	Phobos	7.6 hrs	9,400 km	**6.4 x 10^{23} kg**
Sun	Earth	365 days	149 million km	**1.9 x 10^{30} kg**
Sun	Neptune	163.7 yrs	4.5 million km	**2.1 x 10^{30} kg**
Sirius A	Sirius B	50.1 yrs	298 million km	**6.6 x 10^{30} kg**
Polaris A	Polaris B	30.5 yrs	453 million km	**6.2 x 10^{28} kg**
Milky Way	Sun	225 million yrs	26,000 light years	**1.7 x 10^{41} kg**

Note: The masses for Sirius A and Polaris A are estimates because the companion star has a mass nearly equal to the primary so that our mass formula becomes less reliable.

Problem Solving Using Algebraic Models



The Moon has a mass of 7.4×10^{22} kilograms and a radius of 1,737 kilometers. Seismic data from the Apollo seismometers also shows that there is a boundary inside the Moon at a radius of about 400 kilometers where the rock density or composition changes. Astronomers can use this information to create a model of the Moon's interior. The density of a planet D = M/V where M is its total mass and V is its total volume.

Problem 1 - What is the average density of the Moon in grams per cubic centimeter (g/cm^3) ? (Assume the Moon is a perfect sphere.)

Problem 2 - What is the volume, in cubic centimeters, of A) the Moon's interior out to a radius of 400 km? and B) The remaining volume out to the surface?

You can make a simple model of a planet's interior by thinking of it as an inner sphere (the core) with a radius of R(core), surrounded by a spherical shell (the mantle) that extends from R(core) to the planet's surface, R(surface). We know the total mass of the planet, and its radius, R(surface). The challenge is to come up with densities for the core and mantle and R(core) that give the total mass that is observed.

Problem 3 - From this information, what is the total mass of the planet model in terms of the densities of the two rock types (D1 and D2) and the radius of the core and mantle regions R(core) and R(surface)?

Problem 4 - The densities of various rock types are given in the table below.

Type	Density
I - Iron Nickel mixture (Earth's core)	15.0 gm/cc
E - Earth's mantle rock (compressed)	4.5 gm/cc
B - Basalt	2.9 gm/cc
G - Granite	2.7 gm/cc
S - Sandstone	2.5 gm/cc

A) How many possible lunar models are there? B) List them using the code letters in the above table, C) If denser rocks are typically found deep inside a planet, which possibilities survive? D) Find combinations of the above rock types for the core and mantle regions of the lunar interior model, that give approximately the correct lunar mass of 7.4×10^{25} grams. [Hint: use an *Excel* spread sheet to make the calculations faster as you change the parameters.] E) If Apollo rock samples give an average surface density of 3.0 gm/cc, which models give the best estimates for the Moon's interior structure?

Answer Key

Problem 1 - Mass = 7.4×10^{22} kg x 1000 gm/kg = 7.4×10^{25} grams. Radius = 1,737 km x 100,000 cm/km = 1.737×10^{8} cm. Volume of a sphere = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (1.737 \times 10^8)^3 = 2.2 \times 10^{25}\ cm^3$, so the density = 7.4×10^{25} grams / $2.2 \times 10^{25}\ cm^3$ = **3.4 gm / cm^3**.

Problem 2 - A) V(core) = $4/3\ \pi\ R^3 = 4/3 \times (3.141) \times (4.0 \times 10^7)^3$ = **$2.7 \times 10^{23}\ cm^3$**
B) V(shell) = V(Rsurface) - V(Rcore) = $2.2 \times 10^{25}\ cm^3 - 2.7 \times 10^{23}\ cm^3$ = **$2.2 \times 10^{25}\ cm^3$**

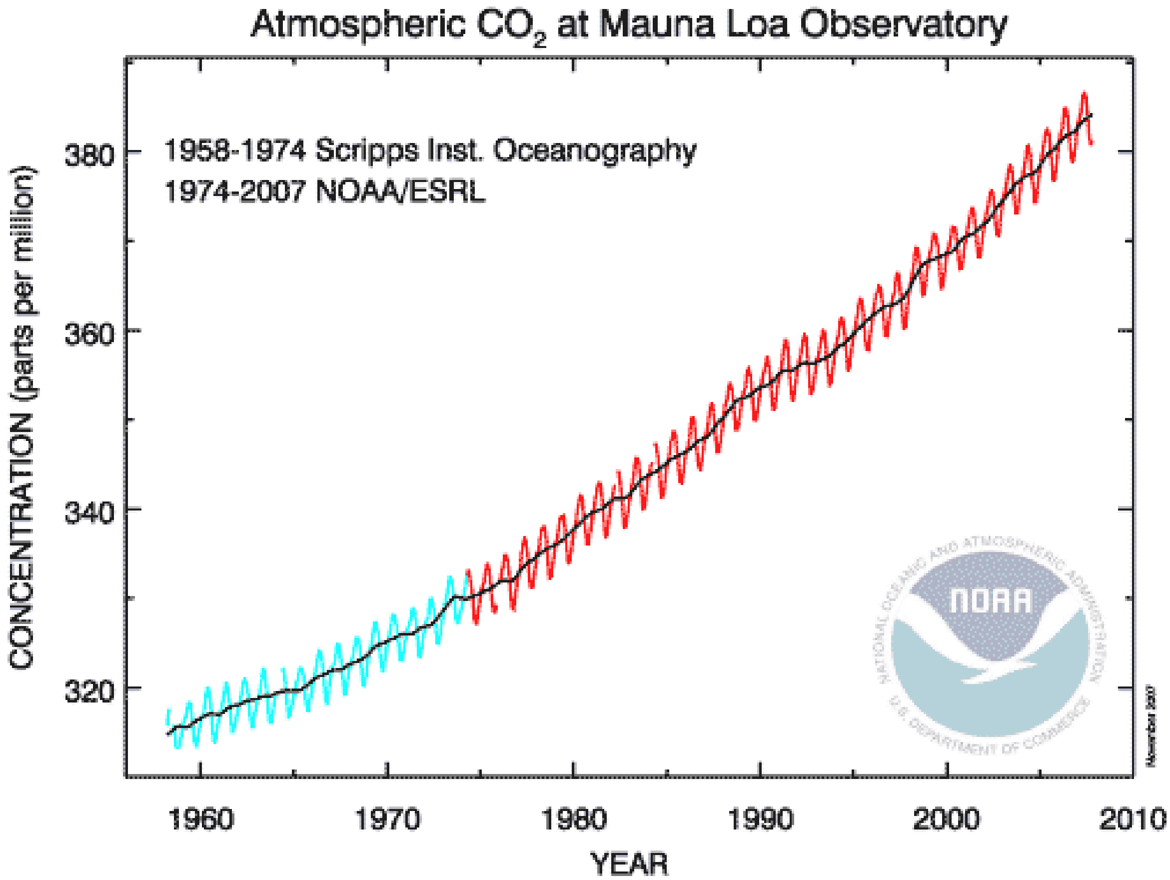
Problem 3 - The total core mass is given by M(core) = $4/3\ \pi\ (Rcore)^3 \times D1$. The volume of the mantle shell is given by multiplying the shell volume V(shell) calculated in Problem 2B by the density : Mshell = V(shell) x D2. Then, the formula for the total mass of the model is given by: $MT = 4/3\ \pi\ (Rc)^3 \times D1 + (4/3\ \pi\ (Rs)^3 - 4/3\ \pi\ (Rc)^3) \times D2$, which can be simplified to:

$$\mathbf{MT = 4/3\ \pi\ (D1 \times Rc^3 + D2 \times Rs^3 - D2 \times Rc^3)}$$

Problem 4 - A) There are 5 types of rock for 2 lunar regions so the number of unique models is 5 x 5 = 25 possible models. B) The possibilities are: II, IE, IB, IG, IS, EE, EI, EB, EG, ES, BI, BE, BB, BG, BS, GI ,GE, GB, GG, GS, SI, SE, SB, SG, SS. C) The ones that are physically reasonable are: IE, IB, IG, IS, EB, EG, ES, BG, BS, GS. The models, II, EE, BB, GG and SS are eliminated because the core must be denser than the mantle. D) Each possibility in your answer to Part C has to be evaluated by using the equation you derived in Problem 3. This can be done very efficiently by using an Excel spreadsheet. The possible answers are as follows:

Model Code	Mass (in units of 10^{25} grams)
I E	10.2
I B	6.7
E B	6.4
I G	6.3
E G	6.0
B G	6.0
I S	5.8
E S	5.5
B S	5.5
G S	5.5
Actual moon composition	**7.4**

E) The models that have rocks with a density near 3.0 gm/cc as the mantle top layer are the more consistent with the density of surface rocks, so these would be IB and EB which have mass estimates of 6.7×10^{25} and 6.4×10^{25} grams respectively. These are both very close to the actual moon mass of 7.4×10^{25} grams (e.g. 7.4×10^{22} kilograms) so it is likely that the moon has an outer mantle consisting of basaltic rock, similar to Earth's mantle rock (4.5 gm/cc) and a core consisting of a denser iron+nickel mixture (15 gm/cc).



This is the Keeling Curve, derived by researchers at the Mauna Kea observatory from atmospheric carbon dioxide measurements made between 1958 - 2005. The accompanying data in Excel spreadsheet form for the period between 1982 and 2008 is provided at

http://spacemath.gsfc.nasa.gov/data/KeelingData.xls

Problem 1 - Based on the tabulated data, create a single mathematical model that accounts for, both the periodic seasonal changes, and the long-term trend.

Problem 2 - Convert your function, which describes the carbon dioxide volume concentration in parts per million (ppm), into an equivalent function that predicts the mass of atmospheric carbon dioxide if 383 ppm (by volume) of carbon dioxide corresponds to 3,000 gigatons.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm) and mass for the years: A) 2020? B)2050, C)2100?

Answer Key

Data from: C. D. Keeling, S. C. Piper, R. B. Bacastow, M. Wahlen, T. P. Whorf, M. Heimann, and H. A. Meijer, Exchanges of atmospheric CO_2 and $^{13}CO_2$ with the terrestrial biosphere and oceans from 1978 to 2000. I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, 88 pages, 2001. Excel data obtained from the Scripps CO2 Program website at http://scrippsco2.ucsd.edu/data/atmospheric_co2.html

Problem 1 - Answer: The general shape of the curve suggests a polynomial function of low-order, whose amplitude is modulated by the addition of a sinusoid. The two simplest functions that satisfy this constraint are a 'quadratic' and a 'cubic'… where 't' is the elapsed time in years since 1982

F1 = A sin (Bt + C) + (Dt2 + Et + F) and F2 = A sin(Bt + C) + (Dt3 + Et2 + Ft + G)

We have to solve for the two sets of constants A, B, C, D , E, F and for A, B, C, D, E, F, G. Using *Excel* and some iterations, as an example, the constants that produce the best fits appear to be: F1: (3.5, 6.24, -0.5, +0.0158, +1.27, 342.0) and F2: (3.5, 6.24, -0.5, +0.0012, -0.031, +1.75, +341.0). Hint: Compute the yearly averages and fit these, then subtract this polynomial from the actual data and fit what is left over (the residual) with a sin function.) The plots of these two fits are virtually identical. We will choose Fppm = F1 as the best candidate model because it is of lowest-order. The comparison with the data is shown in the graph below: red=model, black=monthly data. Students should be encouraged to obtain better fits.

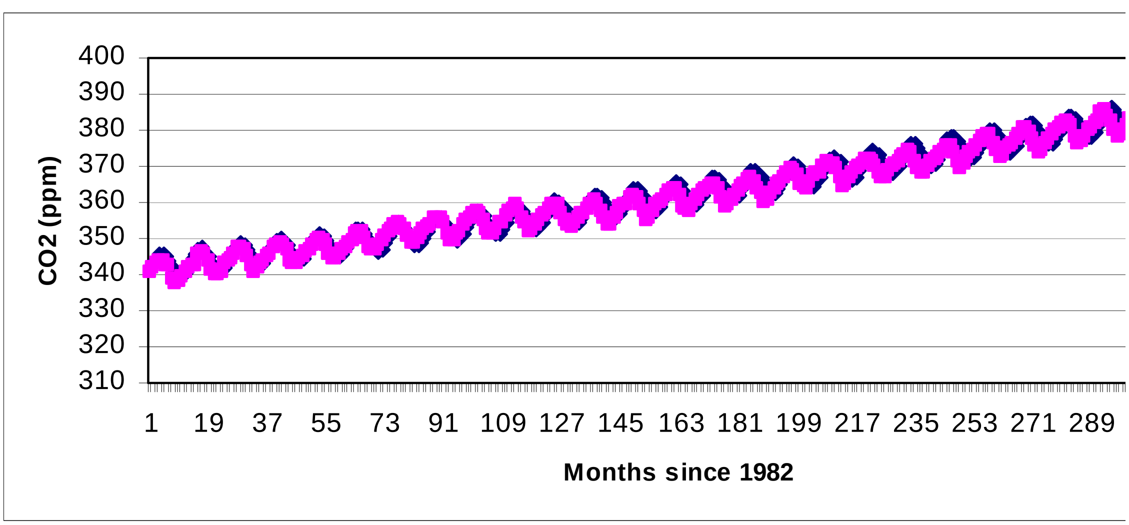
Problem 2 - Answer: The model function gives the atmospheric carbon dioxide in ppm by volume. Since 3000 gigatons = 383 ppm, take Fppm and multiply it by the conversion factor (3,000/383) = 7.83 gigatons/ppm to get the desired function, Fco2 for the carbon mass.

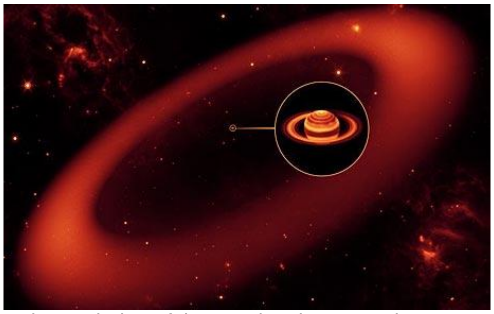
Problem 3 - What would you predict as the carbon dioxide concentration (ppm), and mass for the years: A) 2020? B)2050, C)2100? Answer:

A) t = 2020-1982 = 38, so Fco2(38) = 7.83 x **410 ppm = 3,200 gigatons**

B) t = 2050-1982 = 68, so Fco2(68) = 7.83 x **502 ppm = 3,900 gigatons**

C) t = 2100-1982 = 118, so Fco2(118) = 7.83 x **718 ppm = 5,600 gigatons**




Artist rendering of the new ice ring around Saturn detected by the Spitzer Space Telescope.

"This is one supersized ring," said one of the authors, Professor Anne Verbiscer, an astronomer at the University of Virginia in Charlottesville. Saturn's moon Phoebe orbits within the ring and is believed to be the source of the material.

A thin array of ice and dust particles lies at the far reaches of the Saturnian system. The ring was very diffuse and did not reflect much visible light but the infrared Spitzer telescope was able to detect it. Although the ring dust is very cold -273 C it shines with thermal 'heat' radiation. No one had looked at its location with an infrared instrument until now.

"The bulk of the ring material starts about 6.0 million km from the planet, extends outward about another 12 million km, and is 2.6 million km thick. The newly found ring is so huge it would take 1 billion Earths to fill it." (CNN News, October 7, 2009)

Many news reports noted that the ring volume was equal to 1 billion Earths. Is that estimate correct? Let's assume that the ring can be approximated by a washer with an inner radius of r, an outer radius of R and a thickness of h.

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Problem 3 - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Problem 6 - How does your answer compare to the Press Release information? Why are they different?

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Answer: The area of the large circle is given by πR^2 minus area of small circle πr^2 equals $\mathbf{A = \pi (R^2 - r^2)}$

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Answer: Volume = Area x height so $\mathbf{V = \pi (R^2 - r^2) h}$

Problem 3 - If r = 6×10^6 kilometers, R = 1.2×10^7 kilometers and h = 2.4×10^6 kilometers, what is the volume of the new ring in cubic kilometers?

Answer: $V = \pi (R^2 - r^2) h$

$= (3.141) [(1.2 \times 10^7)^2 - (6.0 \times 10^6)^2] 2.4 \times 10^6$

` $= \mathbf{8.1 \times 10^{20}\ km^3}$

Note that the smallest number of significant figures in the numbers involved is 2, so the answer will be reported to two significant figures.

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Answer: Volume of a sphere $V = 4/3 \pi R^3$ so for Earth,

$V = 1.33 \times (3.14) \times (6.378 \times 10^3)^3$

$= \mathbf{1.06 \times 10^{12}\ km^3}$

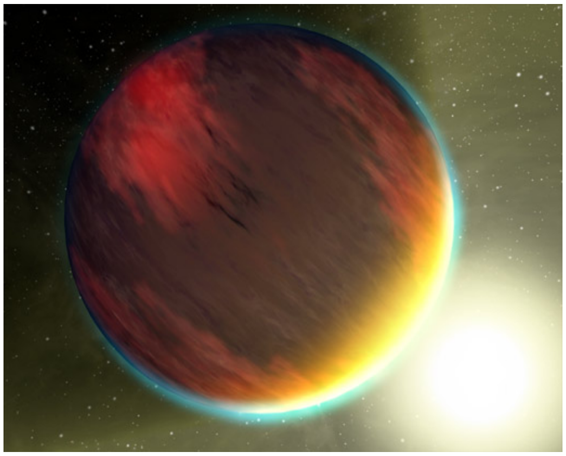
Note that the smallest number of significant figures in the numbers involved is 3, so the answer will be reported to three significant figures.

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Answer: Divide the answer for Problem 3 by Problem 4 to get

$8.1 \times 10^{20}\ km^3 / (1.06 \times 10^{12}\ km^3) = \mathbf{7.6 \times 10^8}$ **times**

Problem 6 - How does your answer compare to the Press release information? Why are they different? Answer: **The Press Releases say 'about 1 billion times' because it is easier for a non-scientist to appreciate this approximate number. If we rounded up $\mathbf{7.6 \times 10^8}$ times to one significant figure accuracy, we would also get an answer of '1 billion times'.**



The basic ingredients for life have been detected in a second hot gas planet, HD 209458b, shown in this artist's illustration. Two of NASA's Great Observatories – the Hubble Space Telescope and Spitzer Space Telescope, yielded spectral observations that revealed molecules of carbon dioxide, methane and water vapor in the planet's atmosphere. HD 209458b, bigger than Jupiter, occupies a tight, 3.5-day orbit around a sun-like star about 150 light years away in the constellation Pegasus. (NASA Press release October 20, 2009)

Some Interesting Facts: The distance of the planet from the star HD209458 is 7 million kilometers, and its orbit period (year) is only 3.5 days long. At this distance, the temperature of the outer atmosphere is about 1,000^{o} C (1,800^{o} F). At these temperatures, water, methane and carbon dioxide are all in gaseous form. It is also known to be losing hydrogen gas at a ferocious rate, which makes the planet resemble a comet! The planet itself has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter. The unofficial name for this planet is Osiris.

Problem 1 - The mass of Jupiter is 1.9 x 10^{30} grams. The radius of Jupiter is 71,500 kilometers. A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere? B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?

Problem 2 - From the information provided; A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere? B) What is the mass of Osiris in grams? C) If Density = mass/volume, what is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Problem 1 - The mass of Jupiter is 1.9×10^{30} grams. The radius of Jupiter is 71,500 kilometers.
A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere?
Answer: The radius of Jupiter, in centimeters, is
R = 71,500 km x (100,000 cm/1 km)
$= 7.15 \times 10^{9}$ cm.

For a sphere, $V = 4/3\, \pi R^3$ so the volume of Jupiter is
$V = 1.33 \times (3.141) \times (7.15 \times 10^{9})^3$
$V = 1.5 \times 10^{30}$ cm^3

B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?
Answer: Density = Mass/Volume so the density of Jupiter is D = (1.9×10^{30} grams)/(1.53×10^{30} cm^3) = **1.2 gm/cc**

Problem 2 - From the information provided;
A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere?
Answer: The information says that the volume is 146% greater than Jupiter so it will be V = 1.53×10^{30} cm^3 (146%/100%)
= **2.2×10^{30} cm^3**

B) What is the mass of Osiris in grams?
Answer: the information says that it is 69% of Jupiter so
M = 0.69 x (1.9×10^{30} grams)
= 1.3×10^{30} grams
C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?
Answer: D = Mass/volume
= 1.3×10^{30} grams / 2.23×10^{30} cm^3
= 0.58 grams/cc

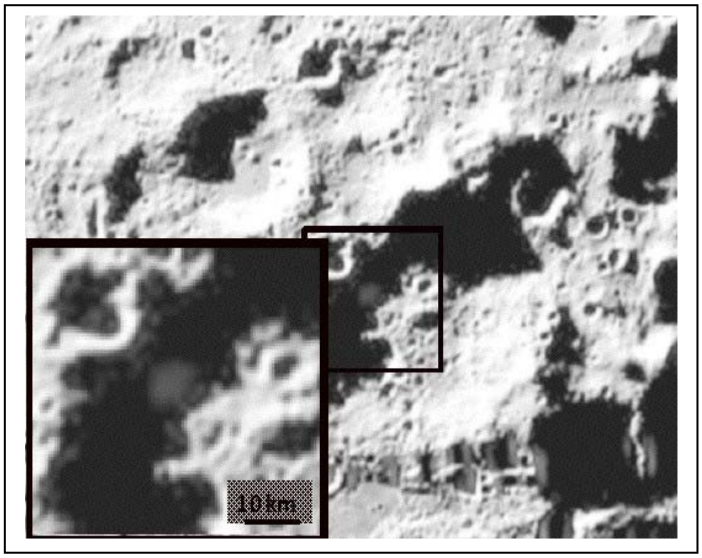
Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Answer: Because the density of Osiris is only about 0.6 grams/cc, the closest match would be **a mixture of hydrogen and helium.** This means that, rather than a solid planet like earth, which is a mixture of higher-density materials such as iron, rock and water, Osiris has much in common with Jupiter which is classified by astronomers as a Gas Giant!

Problem Solving Using Algebraic Models



On October 9, 2009 the LCROSS spacecraft and its companion Centaur upper stage, impacted the lunar surface within the shadowed crater Cabeus located near the moon's South Pole. The Centaur impact speed was 9,000 km/hr with a mass of 2.2 tons.

The impact created a crater about 20 meters across and about 3 meters deep. Some of the excavated material formed a plume of debris visible to the LCROSS satellite as it flew by. Instruments on LCROSS detected about 25 gallons of water.

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Problem 2 - If density = mass/volume, and the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket, what was the concentration of water in the plume in units of tons of regolith per liter of water?

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Answer: V = (3.14) x (10 meters)^2 x 3 meters = **942 cubic meters**.

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Answer: 3000 kg/m^3 x (942 meters^3) = 2,800,000 kilograms. Since 1000 kg = 1 ton, there were **2,800 tons of regolith excavated**.

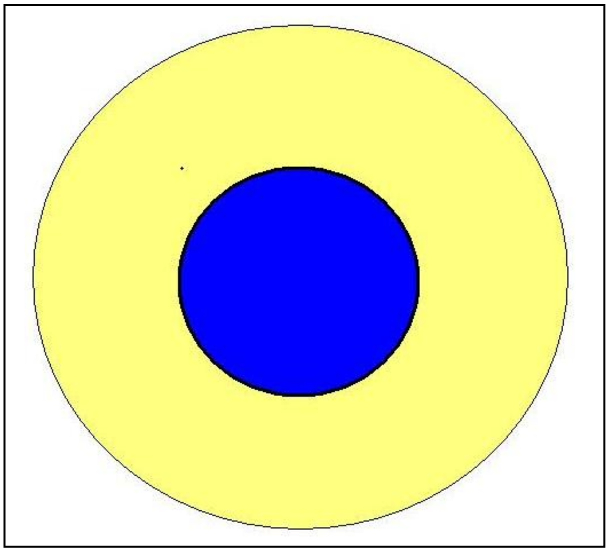
Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Answer: The area of the ejecta blanket is given by A = π(35 meters)^2 - π(10 meters)^2 = 3,846 - 314 = 3500 meters^2. The volume is A x h = (3500 meters^2) x 0.2 meters = 700 meters^3 . Then the mass is just M = (700 meters^3) x (3,000 kilograms/meter^3) = 2,100,000 kilograms or **2,100 tons in the ejecta blanket**.

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Answer: The amount of ejected regolith was 2,800 tons - 2,100 tons or 700 tons. The detected water amounted to 25 gallons or 25 gallons x (3.78 liters/ 1 gallon) = 95 liters. So the concentration was about C = 700 tons/95 liters = **7 tons/liter**.

Note to teacher: The estimated concentration, C, in Problem 4 is based on an approximated geometry for the crater (cylinder), an average thickness for the ejecta blanket (about 0.2 meters) and whether all of the remaining material (700 tons) was actually involved in the plume measured by LCROSS. Students may select, by scaled drawing, other geometries for the crater, and thickness for the ejecta blanket to obtain other estimates for the concentration, C. The scientific analysis of the LCROSS data may eventually lead to better estimates for C.



The planet Osiris orbits 7 million kilometers from the star HD209458 located 150 light years away in the constellation Pegasus. The Spitzer Space Telescope has recently detected water, methane and carbon dioxide in the atmosphere of this planet. The planet has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter.

By knowing the mass, radius and density of a planet, astronomers can create plausible models of the composition of the planet's interior. Here's how they do it!

Among the first types of planets being detected in orbit around other stars are enormous Jupiter-sized planets, but as our technology improves, astronomers will be discovering more 'super-Earth' planets that are many times larger than Earth, but not nearly as enormous as Jupiter. To determine whether these new worlds are Earth-like, they will be intensely investigated to determine the kinds of compounds in their atmospheres, and their interior structure. Are these super-Earths merely small gas giants like Jupiter, icy worlds like Uranus and Neptune, or are they more similar to rocky planets like Venus, Earth and Mars?

Problem 1 - A hypothetical planet is modeled as a sphere. The interior has a dense rocky core, and on top of this core is a crust consisting of a thick layer of ice. If the core volume is 4.18×10^{12} cubic kilometers and the crust volume is 2.92×10^{13} cubic kilometers, what is the radius of this planet in kilometers?

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) How many Earths could fit inside the core of this hypothetical planet? B) How many Earths could fit inside the crust of this hypothetical planet?

Problem 3 - Suppose the astronomer who discovered this super-Earth was able to determine that the mass of this new planet is 8.3 times the mass of Earth. The mass of Earth is 6.0×10^{24} kilograms. What is A) the mass of this planet in kilograms? B) The average density of the planet in kilograms/cubic meter?

Problem 4 - Due to the planet's distance from its star, the astronomer proposes that the outer layer (crust) of the planet is a thick shell of solid ice with a density of 1000 kilograms/cubic meter. What is the average density of the core of the planet?

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?

Problem 1 - The planet is a sphere whose total volume is given by $V = 4/3\ \pi\ R^3$. The total volume is found by adding the volumes of the core and crust to get $V = 4.18 \times 10^{12} + 2.92 \times 10^{13} = 3.34 \times 10^{13}$ cubic kilometers. Then solving the equation for R we get $R = (3.34 \times 10^{13} / (1.33*3.14))^{1/3}$ = 19,978 kilometers. Since the data are only provided to 3 place accuracy, the final answer can only have three significant figures, and with rounding this equals a radius of R **= 20,000 kilometers.**

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number,
A) How many Earths could fit inside the core of this hypothetical planet?
Answer: V = 4.18×10^{12} cubic kilometers / 1.1×10^{12} cubic kilometers = **4 Earths.**

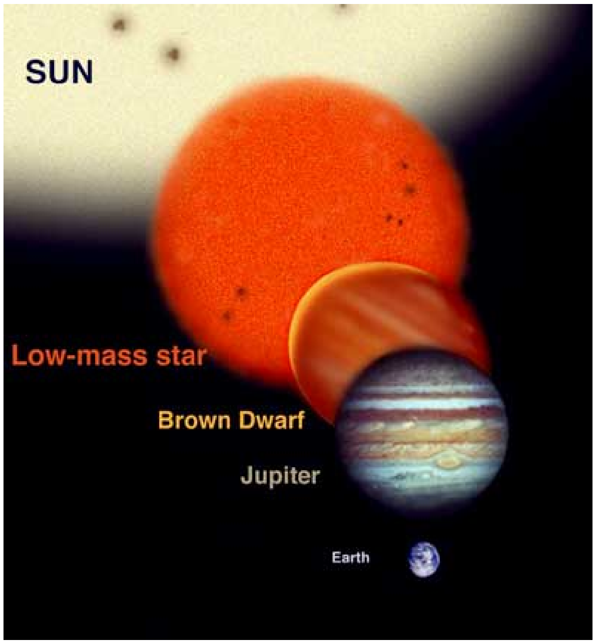
B) How many Earths could fit inside the crust of this hypothetical planet?
Answer: V = 2.92×10^{13} cubic kilometers / 1.1×10^{12} cubic kilometers = **27 Earths.**

Problem 3 - What is A) the mass of this planet in kilograms? Answer: $8.3 \times 6.0 \times 10^{24}$ kilograms **= 5.0×10^{25} kilograms.**
B) The average density of the planet in kilograms/cubic meter?
Answer: Density = total mass/ total volume
= 5.0×10^{25} kilograms/3.34×10^{13} cubic kilometers
= 1.5×10^{12} kilograms/cubic kilometers.
Since 1 cubic kilometer = 10^9 cubic meters,
= 1.5×10^{12} kilograms/cubic kilometers x (1 cubic km/10^9 cubic meters)
= **1,500 kilograms/cubic meter.**

Problem 4 - We have to subtract the total mass of the ice shell from the mass of the planet to get the mass of the core, then divide this by the volume of the core to get its density. Mass = Density x Volume, so the crust mass is 1,000 kg/ $m^3 \times 2.92 \times 10^{13}\ km^3 \times (10^9\ m^3/km^3) = 2.9 \times 10^{25}$ kg. Then the core mass = 5.0×10^{25} kilograms - 2.9×10^{25} kg = 2.1×10^{25} kg. The core volume is $4.18 \times 10^{12}\ km^3 \times (10^9\ m^3/km^3) = 4.2 \times 10^{21}\ m^3$, so the density is D = 2.1×10^{25} kg / $4.2 \times 10^{21}\ m^3$= **5,000 kg/ m^3.**

Problem 5 - The densities of some common ingredients for planets are as follows:
Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3
From your answer to Problem 4, what is the likely composition of the core of this planet?
Answer: **Basalt.**

Note that, although the average density of the planet (1,500 kg/m^3) is not much more than solid ice (1,000 kg/m^3), the planet has a sizable rocky core of higher density material. Once astronomers determine the size and mass of a planet, these kinds of 'shell-core' models can give valuable insight to the composition of the interiors of planets that cannot even be directly imaged and resolved! In more sophisticated models, the interior chemistry is linked to the temperature and location of the planet around its star, and proper account is made for the changes in density inside a planet due to compression and heating. The surface temperature, which can be measured from Earth by measuring the infrared 'heat' from the planet, also tells which compounds such as water can be in liquid form.



During the last 20 years, astronomers have discovered many new types of objects. The most exciting of these are the brown dwarfs. These objects are too small to be stars, but too big to be planets.

(Credit: Gemini Observatory/Artwork by Jon Lomberg)

Whether an object is a star, a brown dwarf, or just a large planet, depends on how it is constructed.

Stars are so massive that they can create their own light by thermonuclear fusion. The minimum mass for this to happen is 80 times the mass of Jupiter (80 MJ).

Planets are small enough that they hold themselves up under the crushing force of gravity just by the compression strength of the materials from which they are formed; mainly rock. They can be either bare-rocky bodies such as Earth, and Mercury, or have dense massive atmospheres of gas overlaying these rocky cores, like Jupiter, Saturn, Uranus and Neptune. The most massive bodies we would recognize as planets have masses below 13 times the mass of Jupiter (13 MJ).

In between the low-mass stars and the high-mass planets is a third category called brown dwarfs. Over 1000 are now known. These bodies are hot enough that a peculiar kind of gas pressure called 'degeneracy pressure' keeps them supported under their own gravity, but even the most massive ones cannot muster the temperatures needed to start thermonuclear fusion.

Problem 1 - Write a linear inequality that summarizes the three mass ranges in the above discussion. If 1 Jupiter mass equals 0.001 times the mass of the sun (0.001 Msun), convert the ranges in terms of Jupiter masses, MJ) into solar masses Msun.

Problem 2 - Classify the following objects: Gliese 581 (M = 0.3 Msun); CFBDSJ005910 (M = 30 MJ); GJ758B (15 MJ) and LHS2397aB (0.068 Msun).

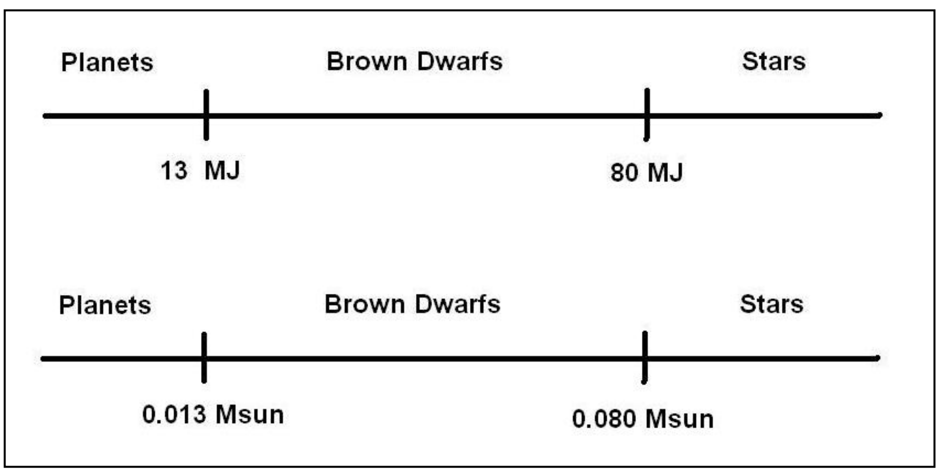
Problem 1 - Write a linear inequality that summarizes the three mass ranges in the above discussion. If 1 Jupiter mass equals 0.001 times the mass of the sun (0.001 Msun), convert the ranges in terms of Jupiter masses, MJ) into solar masses Msun.

Answer:

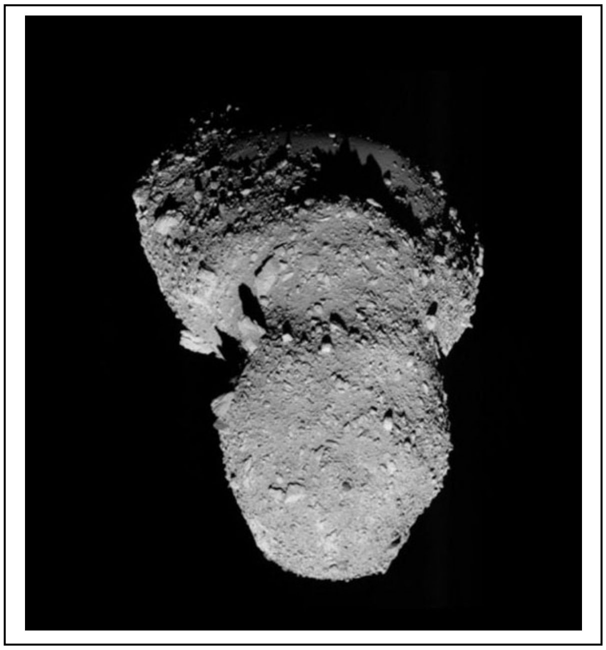
Stars:	M > 80 MJ	becomes	M > 0.08 Msun
Brown Dwarfs;	13 MJ < M < 80 MJ	becomes	0.013 Msun < M < 0.08 Msun
Planets :	M < 13 MJ	becomes	M < 0.013 Msun

Problem 2 - Classify the following objects:
Gliese 581 (M = 0.3 Msun);
CFBDSJ005910 (M = 30 MJ);
GJ758B (15 MJ) and
LHS2397aB (0.068 Msun).

Answer: Gliese 581 is in the mass range for a small **star**.
CFBDSJ005910 is in the mass range for a **brown dwarf**.
GJ758B is in the mass range for a small **brown dwarf**.
LHS2397aB is in the mass range for a **brown dwarf**.



Solving Linear Inequalities: Asteroids



Our solar system is filled by over 100,000 asteroids, comets and planets. Astronomers have studied these bodies, and classified them according to where they are mostly located.

The basic unit of distance measure in our solar system is the Astronomical Unit (AU), which is the distance between the Earth and the Sun (150 million kilometers).

Close-up of Asteroid Itokawa 50 km across. (Courtesy Hayabusha satellite (JAXA)

Kuiper Belt Objects are located between 20 AU and 50 AU from the sun; Oort Cloud bodies are located more than 50 AU from the sun, and the Asteroid Belt is located between 1.3 and 3.5 AU from the sun.

In addition to these small solid bodies, astronomers have discovered over 200 different kinds of comets that fall into two classes: Short Period Comets with orbits between 2 AU and 30 AU, and Long Period Comets with orbits greater than 30 AU.

Problem 1 - Over what range of distances can Short Period Comets be Kuiper Belt Objects?

Problem 2 - What is the range for the Asteroid belt objects in kilometers?

Answer Key

1.6.2

Problem 1 - Over what range of distances can Short Period Comets be Kuiper Belt Objects?

Answer: The various inequalities defining the object categories are:

Oort Cloud	D > 50 AU
Kuiper Belt	20 AU < D < 50 AU
Asteroid Belt	1.3 AU < D < 3.5 AU
Long Period Comets:	D > 30 AU
Short Period Comets:	2 AU < D < 30 AU

For Kuiper Belt Objects and Short Period Comets:

Kuiper Belt	20 AU < D < 50 AU
Short Period Comets:	2 AU < D < 30 AU

So the solution for the overlap is **20 AU < D < 30 AU**

Problem 2 - What is the range for the Asteroid belt objects in kilometers?

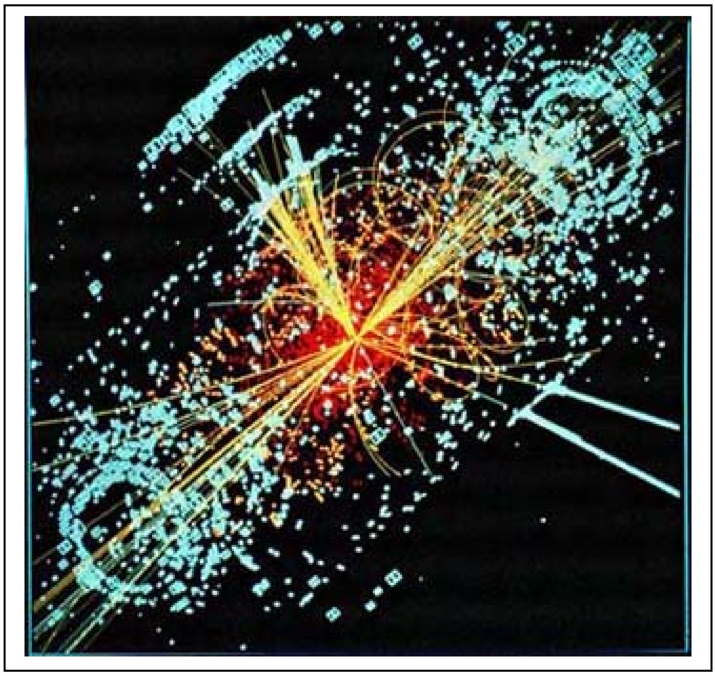
Answer: Since 1 AU = 150 million kilometers:

Asteroid Belt 1.3 AU < D < 3.5 AU

becomes

Asteroid Belt **195 million kilometers < D < 525 million kilometers**

Solving Linear Inequalities: The Higgs Boson



For over 30 years, physicists have been hunting for signs of a new kind of elementary particle. Currently we know about electrons, protons and neutrons among many others.

The Higgs Boson is unique because it is the one kind of particle that actually makes it possible for all other forms of matter to have the property that we call mass.

Although it won't be seen directly, the shower of streaks from the center of the collision of two protons will signal its existence. (Image courtesy of CERN)

There have been many attempts to actually detect this particle. The following paragraph summarizes the constraints on the likely mass range for this still-illusive particle.

Fermilab's Tevatron accelerator experiments concluded that it must either be more massive than 170 GeV or less massive than 160 GeV.

CERN's LEP accelerator concluded after years of searching that the Higgs Boson must be more massive than 115 GeV

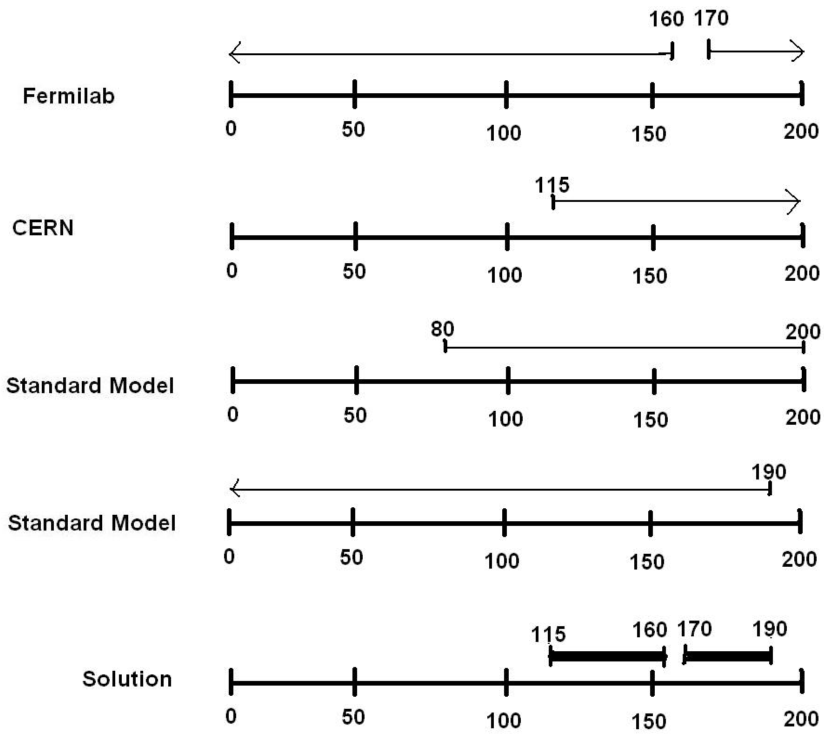
Calculations using the Standard Model, which describes all that is currently known about the interactions between nuclear elementary particles, provided two constraints depending on the particular assumptions used: The Higgs Boson cannot be more massive than 190 GeV, and it has to be more massive than 80 GeV but not more than 200 GeV.

Problem 1 - From all these constraints, what is the intersection of possible masses for the Higgs Boson that is consistent with all of the constraints?

Problem 2 - The mass equivalent to 100 GeV is 1.7×10^{-25} kilograms. What is the mass range for the Higgs Boson in kilograms?

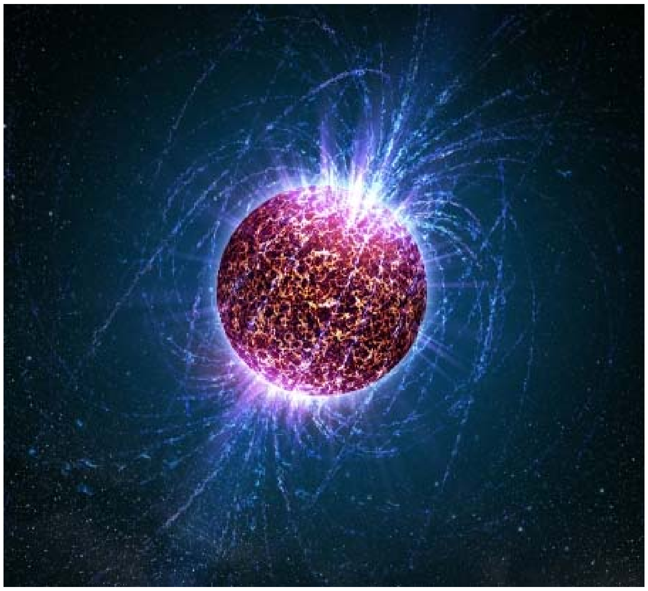
Problem 1 - From all these constraints, what is the remaining range of possible masses for the Higgs Boson that is consistent with all of the constraints?
Answer:

Fermilab Tevatron	M < 160 GeV M > 170 GeV
CERN-LEP	M > 115 GeV
Standard Model	M < 190 GeV
Standard Model	80 GeV < M < 200 GeV



Problem 2 - The mass equivalent to 100 GeV is 1.7×10^{-25} kilograms. What is the mass range for the Higgs Boson in kilograms?
Answer: In GeV the solution is between 115 and 160 GeV or between 170 to 190 GeV. In terms of kilograms this becomes:
1.9×10^{-25} kg to 2.7×10^{-25} kg or between **2.9×10^{-25} kg to 3.2×10^{-25} kg**

Solving Linear Inequalities: Neutron Stars



Neutron stars are all that remains of a massive star that exploded as a supernova. First proposed more than 50 years ago, these dense bodies, barely 50 kilometers in diameter, contain as much mass as our entire sun, which is over 1 million kilometers in diameter.

Astronomers have studied dozens of these dead stars to determine what the mass ranges for neutron stars can be. This mass range is an important clue to understanding what the insides of these bodies looks like.

By studying the x-rays emitted by neutron stars, and by finding many that are in binary star systems, a number of neutron stars have been 'weighed'. Five of these have been measured in detail to compose the following mass ranges, where the mass is given in multiples of the sun's mass (2×10^{30} kilograms):

3U0900-40	$1.2 < M < 2.4$
Centarus X-3	$0.7 < M < 4.3$
SMC X-1	$0.8 < M < 1.8$
Hercules X-1	$0.0 < M < 2.3$

Problem 1 - What is the intersection of these limits for neutron star masses?

Problem 2 - What is the allowed mass range in terms of the mass of a neutron star in kilograms?

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Answer: **$1.2 < M < 1.8$**

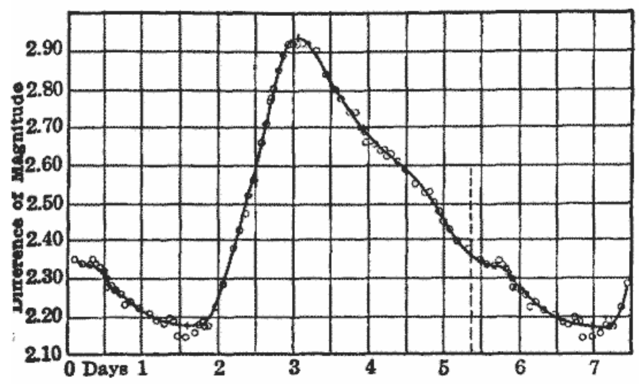
Problem 2 - What is the allowed mass range in terms of the mass of a neutron star in kilograms?

Answer: 1.2 Msun = $1.2 \times 2 \times 10^{30}$ kilograms = 2.4×10^{30} kilograms
1.8 Msun = $1.8 \times 2 \times 10^{30}$ kilograms = 3.6×10^{30} kilograms

So the neutron star mass solution is

2.4×10^{30} kilograms $< M < 3.6 \times 10^{30}$ kilograms

Solving Absolute Value Equations and Inequalities 1.7.1



Brightness change of a Cepheid variable star.

The amount of light produced by a star, measured in watts, can be calculated from the formula

$$L = 9.0\times10^{-16} R^2 T^4$$

in which L is the star's luminosity in units of our sun's total power, R is the radius of the star in units of our sun's radius, and T is the temperature of the star's surface in degrees Kelvin. For example, if the star has 5 times the radius of our sun, and a temperature of 4,000 K, L will be 5.8 times the luminosity of the sun.

Some very old stars enter a phase where they slowly pulsate in size. They expand and cool, then collapse and heat up. The first of these stars studied in detail is called Delta Cephei, located 890 light years from Earth in the constellation Cepheus. Thousands of other 'Cepheid Variable' stars have been discovered over that last 100 years; many of these are located outside the Milky Way in other nearby galaxies.

Delta Cephei increases its radius from 40 times the radius of our sun to 55 times the radius of our sun. At the same time, its temperature cools from 6,800 K to 5,500 K over the course of its cycle.

Problem 1 – Given the above information about the star's changes in radius and temperature, over what range does the luminosity of this star change between its minimum and maximum brightness?

Problem 2 – By what factor does the brightness change between its faintest and brightest levels?

Problem 1 – Given the above information about the star's changes in radius and temperature, over what range does the luminosity of this star change between its minimum and maximum brightness?

Answer: The basic formula is $L = 9.0\times10^{-16} R^2 T^4$

We have $5{,}500 < T < 6{,}800$ and $40 < R < 55$

The maximum value will occur for T = 6,800 K and R = 55 so that
L = 5,800

The minimum value will occur for T = 5,500 K and R = 40 so that
L = 1,300

So the luminosity range is from **1,300 to 5,800** times the luminosity of the sun.

Problem 2 – By what factor does the brightness change between its faintest and brightest levels?

Answer: This is a factor of 5,800/1,300 = 4.5 times between its faintest and brightest levels, which is the change that is observed by astronomers.